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On Large Color Differences in Non-Euclidean Color Spaces

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Summary: The color difference formulas CIE94 and CMC are only applicable to small color differences. For this reason, three papers have been written in which a basis for Euclideanization of these systems and, thus, for the calculation of large color differences was established. The original articles gave the equations for the Riemann spaces that were used to determine by calculus of variation the geodesics for acceptability. Several examples were shown. Subsequently, a direct method of transforming the Riemann space into a Euclidean space was published. With additional calculations, this method could also be applied successfully to the CMC system. This was also proven by examples. Several flaws that surfaced in both systems were listed and corrected (missing upper application limit, missing warning regarding non-Euclideanicity, lack of standardization, missing invariance for the event that reference and sample were transposed).

Introduction

Systems for the measurement of color differences, such as CIELAB and, subsequently, CIE94 and CMC were established as needed for industrial purposes (acceptability). However, they also play a role in colorimetric research (perceptibility). Since these systems are only applicable to the measurement of small color differences, a basis for calculation of larger color differences in the non-Euclidean CIE94 and CMC color spaces was published in three papers [1, 2, 3]. Following is a description of the essential trains of thought that led to these results. The mathematical formulas and programming instructions do not fit the scope of this report. They can be found in the main articles in Farbe [1, 2, 3].

Non-Euclidean Color Spaces

1. CIE94 Color Space

1.1 Calculation Based on Geodesics

While the CIELAB formula expands a Euclidean space in cylinder coordinates, the CIE94 formula [4] has the metrics of a Riemann space, characterized by the so-called line element [1]. The color space is greatly simplified because the long axes of the sectional ellipses in the CIELAB a*,b* plane are always radially oriented in the C^*_{ab}

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direction. In Riemann spaces, the geodesic shown by the integral over the line element, not the straight line, is the shortest distance between two given points. This minimum distance was found with the algorithm of the calculus of variations.

In CIE94, the factors of the CIELAB formula become dependent on the chroma C^*_{ab} of the color locus. This causes the sectional ellipses in the plane to become larger as they move away from the gray axis. Solving and calculating the integrals become more difficult. However, the equations can still be explicitly described. Examples with selected color loci were calculated on the basis of set conditions (see Geodesics, Figures 1 and 2).

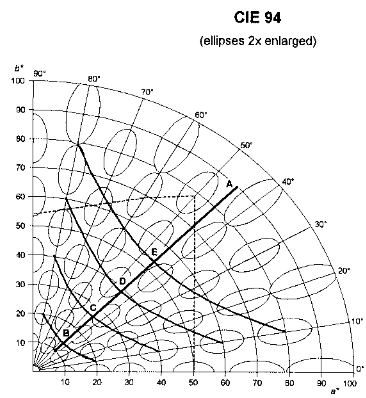

Fig. 1 Color space according to CIE94. Geodesics (1st Series) on the a*,b* plane of the CIELAB color space. The dotted line defines the limit of commercially producible colors for the lightness plane L* = 50 (according to L. GALL [7]).

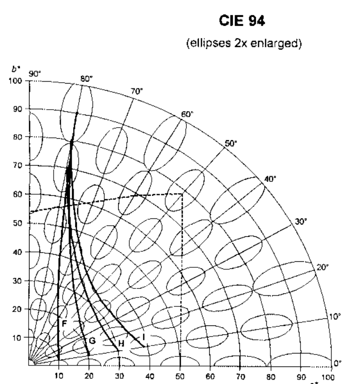

Fig. 2 Color space according to CIE94. Geodesics (2nd Series) on the a*,b* plane of the CIELAB color space. (See Fig. 1 regarding the meaning of the dotted line).

Furthermore, a CIE94(k_L : k_C : k_H) space was shown with the use of ellipsoids, which represent perceptibility as stated by WITT [5]. In this case, the parameter factors k_L, k_C, k_H assume values other than 1 since the basic conditions have been abandoned (see Figure 3). The examples provided for this color space showed the greatest disagreement by far between the color differences calculated with the delta formula and those calculated with the geodesics.

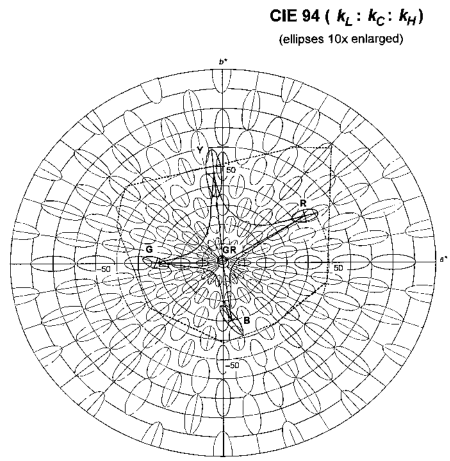

Fig. 3 Color space according to CIE94 (k_L : k_C : k_H). WITT ellipses and geodesics on the a*,b* plane of the CIELAB color space. (See Fig. 1 regarding the meaning of the dotted line).

1.2 Calculation by Euclideanization

Another article then described how the Riemann CIE94 space can be transformed into a Euclidean space [2]. For this purpose, three new coordinates for chroma, hue angle, and lightness were introduced. These were used to calculate the color differences in the known manner. It was found that two coordinates on the a*,b* plane suffice for the calculation.

The results were compared with those obtained on the basis of actual geodesics, and very good agreement was found. In addition, the new formulas were compared with suggestions previously offered by other authors.

2. CMC Color Space

The two publications cited initially show how geodesics in the CIE94 space can be calculated and how the CIE94 space can be transformed into a Euclidean space. This seemed to suggest that the latter method could also be applied to the CMC space [3].

First, obvious flaws in the CMC formula [6] were determined: the lack of standardization and the missing invariance guarding against transposition of the two color loci. Corrective steps were proposed. Subsequently, the line element was established which, at the same time, determines its limiting ellipsoids in the color space. Based on the new Euclidean line element, the new lightness and the new chroma always depend on the old lightness and the old chroma alone. The third coordinate, the new hue angle, was somewhat more difficult to calculate. To accomplish this, the entire color wheel had to be divided into sectors in which different integrals had to be formed.

Then, the color differences were calculated from the new coordinates by the known method. This included the calculation of specific average values for the CIELAB coordinates adapted to the CMC system. The necessary mathematical steps for the graphic presentation were provided. Finally, calculation samples based on examples given in the earlier papers were supplied so that comparisons could be made (see Figures 4 and 5 and Figure 6 after Euclideanization of Figure 5).

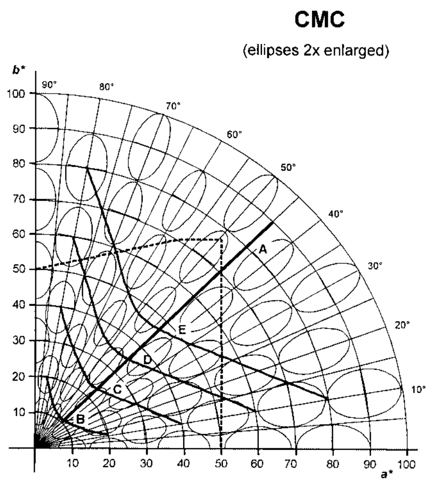

Fig. 4 Color space according to CMC. Geodesics (1st Series) on the a*,b* plane of the CIELAB color space. (See Fig. 1 regarding the meaning of the dotted line).

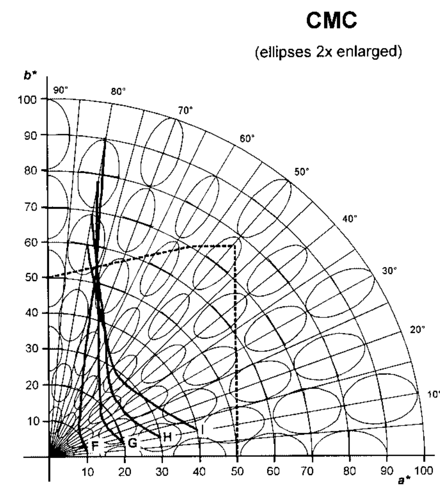

Fig. 5 Color space according to CMC. Geodesics (2nd Series) on the a*,b* plane of the CIELAB color space. (See Fig. 1 regarding the meaning of the dotted line).

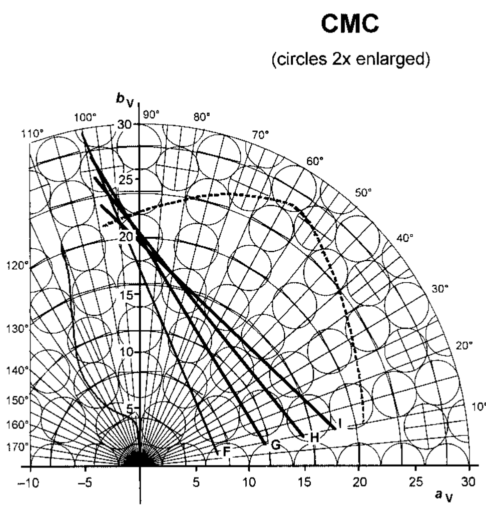

Fig. 6 Color space according to CMC. Geodesics (2nd Series) on the a_V,b_V plane of the CMC color space. (See Fig. 1 regarding the meaning of the dotted line).

Conclusion

In all three papers, flaws in the cited standards were pointed out, and corrective steps were proposed. Regarding CIE94 and CMC this concerns mainly the following:

- An upper application limit must be given.
- A warning concerning non-Euclidicity must be included.
- The formulas must be standardized based on the minimum color difference.

Due to the invariance against transposition of reference and sample, the arithmetical average values of the coordinates must be used.

References

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